CSCI 341 Problem Set 5

From Nonregular Languages to Counter Automata

Due Friday, October 10

Don't forget to check the webspace for hints and additional context for each problem!

Pumping Lengths

Problem 1 (NOT THE Bs)**.** Show that the following language is not regular.

$$L = \{a^n b^m \mid n \in \mathbb{N} \text{ and } n > m\}$$

Solution. □

Context-free Grammars

Problem 2 (Balancing Act)**.** Recall that a string of parentheses is *balanced* if every left parenthese (is eventually followed by a right parenthese). But things get more complicated when there are other alternatives to parentheses: what about square brackets? Or curly ones? If we take

$$A = \{(,), [,], \{,\}, \langle, \rangle\}$$

then we say that a string of brackets $w \in A^*$ is *balanced* if every left bracket of a given type is eventually followed by a right bracket of the same type, without being interrupted by an unmatched right bracket of a different type. For example, these are all balanced:

$$\{()\}() \qquad [] \qquad [()\langle()\rangle] \qquad [()\{()\}] \qquad\qquad (*)$$

but these are not:

$$([)] \qquad \{()() \qquad] \qquad \langle[()()\rangle] \qquad [()\{(\})]$$

Let $L \subseteq A^*$ be the language of balanced strings of brackets.

(1) Write down a grammar $\mathcal{G} = (X, A, R)$ with a variable that generates L, i.e., for some $x \in X$, $\mathcal{L}(\mathcal{G}, x) = L$.

(2) Use your grammar to derive each of the words in (*).

(3) Describe what prevents each of the words in (**) from being derivable from your grammar $\mathcal{G}$.

Solution. □

Problem 3 (Arithmetic is Not Regular)**.** Prove that the language of arithmetic expressions $ArExp \subseteq A^*$, derived from E in the grammar $\mathcal{G} = (X, A, R)$ below

$$E \to N \mid (E + E) \mid (E \times E) \mid (E - E) \mid (E/E)$$
$$N \to 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \mid NN$$

where the alphabet is

$$A = \{(,), +, \times, -, /, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

is not regular.

Solution. □

Parse Trees

Problem 4 (Left on Your Own)**.** Let $\mathcal{G}$ be a grammar with a variable x, and let $w \in A^*$. Prove that if w has a derivation from x, then w has a left-most derivation from x.

Solution. □

Counter Automata

Problem 5 (Cats > Dogs)**.** Let $A = \{c, a, t, d, o, g\}$. Design an integer-counter automaton with a state x that accepts the language L_{cat} of all words $w \in A^*$ such that the string "cat" appears in w more times than "dog" appears in w.