# CSCI 341 Problem Set 2

Language Acceptance; Finite and Infinite Automata; Finitely Recognizable Languages

Due Friday, September 12

## Language Acceptance

**Problem 1** (Cooking with Gas). In each of the following questions, you are asked to design an automaton with a state that accepts a given language. Draw its state diagram and its transition table, and briefly explain why the automaton works.

(1) Over the alphabet $A_1 = \{1, 2\}$ of input letters, define the function $\mathrm{sum}\colon A^* \to \mathbb{N}$ by

$$\mathrm{sum}(\varepsilon) = 0 \qquad \mathrm{sum}(a_1 a_2 \cdots a_n) = a_1 + a_2 + \cdots + a_n$$

So, for example, $\mathrm{sum}(1221) = 1 + 2 + 2 + 1 = 6$. Design an automaton with a state that accepts the language

$$L_1 = \{w \in A^* \mid \mathrm{sum}(w) \text{ is a multiple of } 3\}$$

(2) Over the alphabet $A_2 = \{a, c, t\}$ of input letters, design an automaton with a state that accepts the language

$$L_2 = \{w \in A_2^* \mid w \text{ contains the word } cat\}$$

(3) Over the alphabet $A_3 = A_1 \cup A_2$ of input letters, design an automaton with a state that accepts the language

$$L_3 = L_1 \cdot L_2 = \{wu \in A_3^* \mid w \in L_1 \text{ and } u \in L_2\}$$

**Problem 2** (Pythonic Automaton III). Write a Python script in the same format as the Pythonic Automaton I that implements state $s_1$ in abstract state diagram (1) from the games and puzzles section. Submit your program as a .py file.

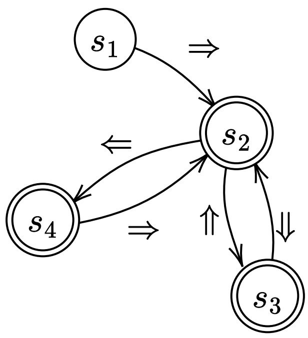

Figure 1: Abstract state diagram (1).

## Finite and Infinite Automata

**Problem 3** (Unravelling an Infinite Language)**.** Draw a state diagram of all of the languages that are reachable from the language $L = \{(ab)^n \mid n \in \mathbb{N}\}$ in the Brzozowski automaton by taking $a$- and $b$-derivatives (the words in this language are $\varepsilon$, $ab$, $abab$, ...). Include all of the double-circled states to indicate which languages are accepting states of the Brzozowski automaton. What language is accepted by $L$?

**Problem 4** (Language Accepts Itself)**.** Let $L \subseteq A^*$ be any language. Prove that $\mathcal{L}(\mathcal{A}_{Brz}, L) \subseteq L$.

## Finitely Recognizable Languages

**Problem 5** (Languages as Trees)**.** Let $A = \{0, 1\}$, and let $L \subseteq A^*$ be a language from $A$. Prove that if $L$ is finite, i.e., $L = \{w_1, \ldots, w_n\}$ for some $n \in \mathbb{N}$, then $L$ is finitely recognizable.

**Problem 6** (Total vs Partial)**.** Prove that $\mathsf{DFin} = \mathsf{TDFin}$ by describing how to turn a deterministic automaton into a total deterministic automaton without changing the languages accepted by the states.